Beispiele zur Logik

beweisbar:

1: $\vdash$ (∃ a. ∀ x. pp(a, x)) → (∀ b. ∃ y. pp(y, b))

1a: $\vdash$ ¬ ∀ a. ¬ ∀ x. pp(a, x) → (∀ b. ∃ y. pp(y, b))

1b: $\vdash$ ¬ ∀ a. ∃ x. ¬ pp(a, x) → (∀ b. ∃ y. pp(y, b))

1c: $\vdash$ ¬ ¬ ∀ a. ∃ x. ¬ pp(a, x) ∨ (∀ b. ∃ y. pp(y, b))

1d: $\vdash$ (∀ a. ∃ x. ¬ pp(a, x)) ∨ (∀ b. ∃ y. pp(y, b))

2: $\vdash$ (∀ x. p(x) ∧ q(x)) → (∀ x. p(x)) ∧ (∀ x. q(x))

2-prop: $\vdash$ (rho → phi) ∧ (rho → psi) ∧ (phi → xi) ∧ (psi → xi) → (rho → xi)

3: $\vdash$ (∃ x. p(x) ∨ q(x)) → (∃ x. p(x)) ∨ (∃ x. q(x))

3a: $\vdash$ (∃ x. ¬ p(x) → q(x)) → (∃ x. p(x)) ∨ (∃ x. q(x))

3c: $\vdash$ (∃ x. p(x) ∨ q(x)) → ((∀ x. ¬ q(x)) → (∃ x. p(x)))

3d: $\vdash$ (∃ x. ¬ q(x) → p(x)) → ((∀ x. ¬ p(x)) → (∃ x. q(x)))

4: $\vdash$ (∀ x. p(x) → qq(x, ff(x))) ∧ p(a) → qq(a, ff(a))

4a: $\vdash$ (∀ x. p(x) → qq(x, ff(x))) ∧ ¬ qq(a, ff(a)) → ¬ p(a)

4b: $\vdash$ (p(a) → qq(a, ff(a))) ∧ ¬ qq(a, ff(a)) → ¬ p(a)

5: $\vdash$ ¬ ((∀ x. p(x)) ∧ (∃ a. ¬ p(a)))

6: $\vdash$ (∀ u. ∃ a. f(a, u)) ∧ (∀ x, y, z. f(x, y) ∧ f(y, z) → gf(x, z)) → (∀ b. ∃ v. gf(v, b))

7: $\vdash$ (∀ x. ff(gg(x)) = x) → ((∀ x. pp(x, ff(x))) → (∀ y. ∃ z. pp(z, y)))

7a: $\vdash$ (∀ x. ∃ y. ff(y) = x) → (¬ ∀ y. ∃ z. pp(z, y) → (∃ x. ¬ pp(x, ff(x))))

7b: $\vdash$ (∀ x. ∃ y. ff(y) = x) → ((∃ y. ¬ ∃ z. pp(z, y)) → (∃ x. ¬ pp(x, ff(x))))

Hilbert-1: $\vdash$ phi → (psi → phi)

Hilbert-2: $\vdash$ (phi → psi) → (¬ psi → ¬ phi)

Hilbert-3: $\vdash$ ((phi → psi) → (phi → xi)) → (phi → (psi → xi))

gf: $\vdash$ (∀ u. ∃ a. a is_f_of u) ∧ (∀ x, y, z. x is_f_of y ∧ y is_f_of z → x is_gf_of z)
→ ∀ b. ∃ v. v is_gf_of b

newex: $\vdash$ p(x) ∨ ¬ p(x)

nh3: (∀ x. p(x) ∨ q(x))
∧ (∀ y. ¬ p(y) ∨ q(ff(y)))
∧ (∀ u. p(u) ∨ ¬ q(u))
∧ (∀ z. ¬ p(z) ∨ ¬ q(ff(z))) $\vdash$

nx01: $\vdash$ (∀ x. p(x) → (∃ y. qq(x, y))) ∧ (∀ z. ¬ qq(a, z)) → ¬ p(a)

s2: ¬ pp(ac, s) ∧ (∀ x, z. pp(x, z) ∨ ¬ q(z)) ∧ (∀ y. q(y) ∨ pp(ac, y)) $\vdash$

nicht beweisbar:

1-oposit-a: $\vdash$ (∀ b. pp(ff(b), b)) → (∃ a. ∀ x. pp(a, x))

1-oposit-b: $\vdash$ (∃ a, b. ff(a) ≠ ff(b) ∧ pp(ff(a), a) ∧ pp(ff(b), b)) ∧ (∀ x. x = a ∨ x = b)
→ ¬ ((∀ b. pp(ff(b), b)) → (∃ a. ∀ x. pp(a, x)))

1-oposit-c: $\vdash$ ff(a) ≠ ff(b) ∧ pp(ff(a), a) ∧ pp(ff(b), b)
→ ¬ ((∀ b. pp(ff(b), b)) → (∃ a. ∀ x. pp(a, x)))

1-oposit-d: $\vdash$ (∀ b. ∃ y. pp(y, b)) → (∃ a. ∀ x. pp(a, x))

3b: $\vdash$ (∃ x. p(x) ∨ q(x)) → ((∀ x. ¬ q(x)) → (∀ x. p(x)))